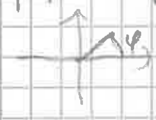
Rashba - Bychkov : 2DEG in (x,y)-plane $\mathcal{U}(z) \to E_z$

$$H = \frac{p^2}{2m} + \alpha \sigma \cdot (\hat{z} \times \vec{p}) = \frac{p^2}{2m} + \alpha(\sigma_y p_x - \sigma_x p_y)$$

Rashba parameter : $\alpha = \frac{e\hbar^2}{4m^2c^2} E_z$ (6 orders of magnitude off $\to$ Coulomb V)
(experimental observables)

Eigenenergies : $\vec{k}_{\parallel} = (k_x, k_y, 0) = k_{\parallel}(\cos\varphi, \sin\varphi, 0)$



$$\mathcal{E}^{\pm}(k_{\parallel}) = \frac{\hbar^2 k_{\parallel}^2}{2m} \pm \alpha k_{\parallel} + \mathcal{E}_0$$

Eigenfunctions : $\psi^{\pm} = \frac{1}{\sqrt{2}} e^{i k_{\parallel} \cdot x} \begin{pmatrix} \pm i e^{i\varphi} \\ 1 \end{pmatrix}$ Eigenspinor

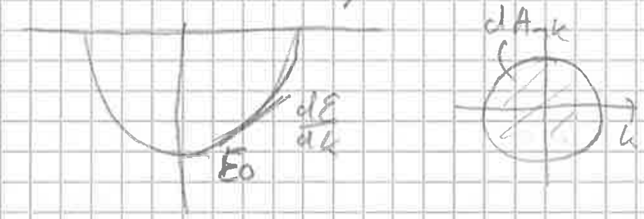
Spin Polarisation : $\bar{P} = \langle \sigma \rangle$ (normalised to 1)

$$\langle \psi^{\pm} | \sigma | \psi^{\pm} \rangle = \begin{pmatrix} \pm \sin\varphi \\ \mp \cos\varphi \\ 0 \end{pmatrix}$$

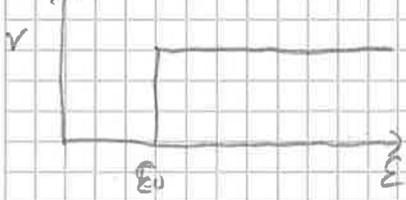
No net P (and M)

* Energy splitting :
$\Delta\mathcal{E} = \mathcal{E}^+ - \mathcal{E}^- = 2\alpha |k_{\parallel}|$ (linear in $k_{\parallel}$)
* Momentum splitting : $\Delta k = 2k_0 = \frac{2\alpha m^*}{\hbar^2}$
* Density of states : $\nu(E) \propto \frac{1}{dA} \frac{dN}{dk} \frac{dk}{dE}$ $\leftarrow$ slope^{-1}
area of contour (dA)

Normal 2D e^- system



$N_{2D} \propto k_{\parallel}^2$ $dA \propto k_{\parallel}^2$
slope^{-1} $\propto \frac{1}{k_{\parallel}}$
$\Longrightarrow$ constant
$\nu(\mathcal{E})_{2D} = \frac{m^*}{\pi\hbar^2}$

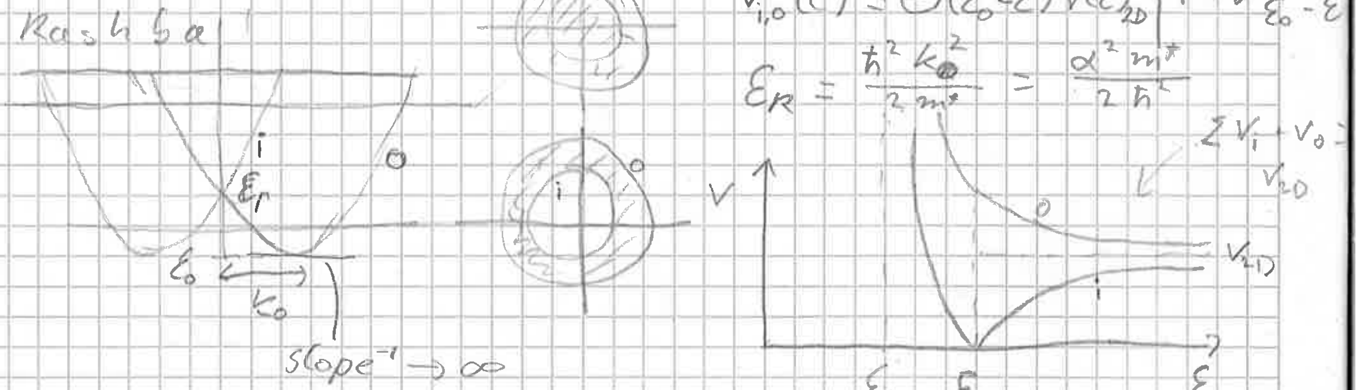


Heaviside step function

$$\nu_{1,0}(\mathcal{E}) = \Theta(\mathcal{E}_0 - \mathcal{E})\, \nu(\mathcal{E})_{2D} \left| 1 \pm \sqrt{\frac{\mathcal{E}_R}{\mathcal{E}_0 - \mathcal{E}}} \right|$$

$$\mathcal{E}_R = \frac{\hbar^2 k_0^2}{2m^*} = \frac{\alpha^2 m^*}{2\hbar^2}$$

Rashba



slope$^{-1} \to \infty$
and $dA \not\to 0$ (finite)

$\Longrightarrow$ singularity in $\nu(\mathcal{E})$

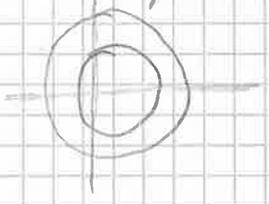
Van Hove singularity (generalise)

Rashba-based spin filter

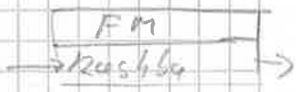
* $E_0 < E_i < E_R$: all states with $k_1 > 0$ same spin.
  Not ideal spin filter: group velocity and k decoupled!
* At E_R $v_1 = 0$ $\Longrightarrow$ spin filter (also base for Majorana)

Spin-orbit torque

* In equilibrium spin balanced $(M = 0)$



In plane voltage → shift Fermi surface
$\Longrightarrow$ spin imbalance → temporary $M \neq 0$
$\Longrightarrow$ switch magnetic layer



(Edelstein: spin ⟷ charge)

Spin transistor + spin algebra

Spinors: (z as basis)

$$\varphi_{z+} = \begin{pmatrix}1\\0\end{pmatrix} \quad \varphi_{z-} = \begin{pmatrix}0\\1\end{pmatrix}$$
$$\varphi_{y+} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} \quad \varphi_{y-} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$
$$\varphi_{x+} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad \varphi_{x-} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Example:

$$\varphi_{z+} + \varphi_{z-} = \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} = \sqrt{2}\,\varphi_{x+}$$
$$\varphi_{z+} + e^{i\pi/2}\varphi_{z-} = \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\i\end{pmatrix} = \begin{pmatrix}1\\i\end{pmatrix} = \sqrt{2}\,\varphi_{y+}$$

* Inject φ_{z+} in Rashba → superposition of $\varphi_{y\pm}$ →
  accumulate phase $\Delta\Phi = 2k_0 L$ (L travel length) →
  spin rotates in (x, z) plane ($180°$ rotation after $L_R = \pi/k_0$)